N62-16293

See N62-12869

NASA TN D-1421

NASA TN D-1421

TECHNICAL NOTE

D-1421

PULSE FREQUENCY MODULATION

R. W. Rochelle

Goddard Space Flight Center
Greenbelt, Maryland

OTS PRICE

XEROX $ 1.10ph

MICROFILM $ 0.80mf

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON October 1962

N62 - 16293

PULSE FREQUENCY MODULATION

by
R. W. Rochelle
Goddard Space Flight Center

SUMMARY

Pulse frequency modulation has been successfully employed as the encoding technique in a number of small United States earth satellites where reduction of power and weight are prime considerations. This paper introduces some of the more basic principles of pulse frequency modulation. A comparison is made to the better known characteristics of coded binary sequences to prove the orthogonality of this type of modulation. It is shown that pulse frequency modulation with quantized frequencies has the same communication efficiency in the presence of additive white Gaussian noise as a corresponding set of coded binary sequences with an equal number of quantized levels.

CONTENTS

PULSE FREQUENCY MODULATION*

by
R. W. Rochelle
Goddard Space Flight Center

INTRODUCTION

The telemetering of data from satellites and space probes requires highly efficient communication systems if maximum use of payload weight is to be realized. The use of such systems tends to increase the complexity and, therefore, reduce the overall reliability of the data acquisition process. A useful spacecraft telemeter is one that is as efficient as possible consistent with good reliability. Pulse frequency modulation (PFM) was developed with these two criteria in mind. It takes on the characteristics of an optimally encoded signal, and employs a minimum number of components to perform the encoding functions.

The first United States earth satellite carried FM/PM and FM/AM telemetry systems. Although these, as used, were not high-efficiency systems, they did have the virtue of reliability. This reliability was partly attained by the experience gained from their use in many missile and rocket flights.

The first digital system was carried into orbit by Explorer VI (1959δ2). The system, called Telebit, employed uncoded binary pulse-code modulation with a variable bit-rate. In September, 1959, Vanguard III (1959 η) was launched carrying a combination of pulse duration modulation and pulse frequency modulation. Similar systems were carried by Explorer VIII (1960 ξ) in November, 1960, and by Explorer X (1961 κ) in March, 1961. Explorer X measured the interplanetary magnetic field out to a distance of 150,000 miles from the earth. On August 15, 1961, Explorer XII (1961 υ) carried the first true pulse frequency modulation system into a highly elliptical orbit with an apogee of approximately 50,000 miles. A number of other satellites, either under construction or contemplated, are scheduled to use PFM.

*Published in *IRE Trans. on Space Electronics and Telemetry*, SET-8(2):107-111, June 1962

GENERAL DESCRIPTION

Pulse frequency modulation combines some of the better features of binary pulse code modulation with the analog features of frequency modulation. Both digital and analog signal inputs are easily encoded. Fortuitously, the signal exhibits some of the characteristics of an optimally encoded signal, allowing a decrease in the energy required per bit for the same error-rate probability. A more important advantage, and one which has been capitalized upon in a number of satellite systems, is that the analog signal can be quantized on the ground after reception; this results in the absence of quantizing noise when the signal-to-noise ratio is good.

Pulse frequency modulation has had a number of other names in the past. Satellite command systems which use one or more tone bursts as codes to start the command functions are examples of PFM systems. Pulse radars whose returns are shifted in frequency because of Doppler effects employ PFM techniques in the detection process. Wolf (Reference 1) has analyzed multiple-tone frequency-shift-keyed transmission systems and has shown that these are orthogonal systems. Kotel'nikov (Reference 2) has analyzed this type of signal and calculated its immunity to noise.

Little if any advantage was gained by using PFM as an encoding technique in communication systems until the advent of the combination of the transistor and magnetic core. The low power drain, light weight, and long life made the transistor an ideal component for spacecraft electronics. The magnetic core with square-hysteresis-loop material allowed the construction of an oscillator capable of starting in one-half cycle of its oscillating frequency; this is a necessity for pulsed operation.

A typical pulse frequency modulation signal consists of a series of sequential pulses; the frequency of each pulse conveys the information and is proportioned to the amplitude of one of the input signals. Figure 1 shows the format used for a typical PFM signal. If there are sixteen channels of information to be telemetered, each channel is associated with a separate pulse, and the frequencies of the sixteen pulses resulting would represent

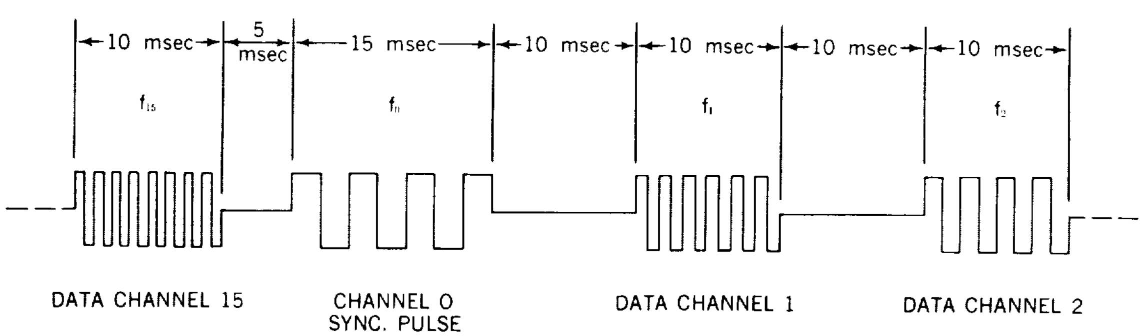

Figure 1—Pulse frequency modulation telemetry format

the sequential sampling of the sixteen channels. In practice, one channel would be reserved for synchronization, leaving the remaining fifteen channels available for time-multiplexing the information. In the case of Explorer XII, the duration of the pulse frequency was 10 msec, and the time between pulses was also 10 msec to allow sufficient recovery time for the comb-filters used in the detection process.

In most space telemetry applications, the information to be transmitted is in both analog and digital forms. Input signals in analog form are encoded by PFM in much the same manner as by an FM/FM telemeter. A pulsed subcarrier oscillator is used to convert the 0 to +5 volt input to a pulse frequency output. Again using the Explorer XII case, an input of 0 volts gives a pulse frequency of 15 kc; 2.5 volts gives 10 kc; and 5 volts gives 5 kc. The linearity of the pulsed subcarrier analog oscillator is better than ±1/2 percent over the 3:1 dynamic frequency range. The long term accuracy of measurement is well within 1 percent, while short term precision-type measurements can be made to 1/10 percent.

Information already in a digital form is easily encoded. A pulsed digital oscillator has been developed which accepts three bits of information and encodes this as one of eight discrete frequencies in the band from 5 to 15 kc. To encode digital channels such as the output of a string of counter stages from a radiation-detection experiment, the binary stages of the counter are read out three at a time with the digital oscillator. The three input leads to the digital oscillator are then switched to the next three counter stages, and one of the eight frequencies is produced which is indicative of the next three bits in the counter. Thus, by reading the counter stages three bits at a time, the complexity of the switching circuitry is materially reduced.

By means of these analog and digital oscillators, both analog and digital signals may readily be encoded in the same system. Either type of oscillator may be commutated from channel to channel, or one oscillator may be used for each channel. Since only one oscillator is pulsed on at a time, the addition of more oscillators has no effect on the power drain. All of the oscillators are connected together, through the use of diode logic, in such a way that only the oscillator which is pulsed "on" is connected to the modulator. The modulator is usually a two-transistor saturating amplifier which is transformer-coupled to the transmitter. Since the output of the analog and digital oscillators is a square wave, the saturated modulator output to the transmitter is also a square wave.

The output of the modulator may be used either to amplitude, phase, or frequency modulate the transmitter carrier. Both amplitude and phase modulation have the advantage that the RF signal can be phase-detected coherently. Because of the square-wave nature of the modulating signal, coherent detection is difficult to employ with frequency modulation. Phase modulation is favored in that the ratio of sideband power to carrier power may be adjusted by changing the peak phase deviation. Since range and information rates are different for different spacecraft missions, the carrier power is set to provide adequate tracking information.

THEORY

A message from a spacecraft is generally a composite signal made up from portions of the outputs of the various experiments aboard. If these outputs are encoded together in an efficient manner, the total payload weight can be reduced by the savings realized by the reduction of the transmitter power. Small spacecraft are especially affected by this relation since the transmitter power supply is a sizeable portion of the total payload weight. Choosing the right encoding scheme, then, can either result in increases in information rate, range, and signal-to-noise ratio, or a decrease in the payload weight.

Under the guide lines set down by Shannon in his classic paper (Reference 3), the search continues for a means of realizing the maximum channel capacity:

$$C = W \log_2 \frac{P + N}{N} , \tag{1}$$

where

C = channel capacity (bits/sec),
W = channel bandwidth,
P = signal power, and
N = average noise power.

Various encoding schemes (References 4 and 5) for binary PCM which approach this maximum efficiency have been used and reported in the literature. These either add parity information or choose a group code which has unique properties. Extra bits added in the parity information increase the bandwidth but allow a decrease in the input signal-to-noise ratio by virtue of their error-detecting and error-correcting ability. The group codes are made up of selected binary sequences. This latter class will be further discussed in the following paragraphs, since it will be shown that PFM has the properties of a set of these special group codes.

For a binary PCM signal encoded with n-bit words, there are 2^n different words or sequences of *zeros* and *ones* available for each word. For 4-bit words there are sixteen sequences. Of these sixteen there are two sets of eight sequences which form two unique code groups. If circuitry is devised whereby three bits of information are encoded by one of these groups of eight sequences, an improvement can be made in the signal-to-noise ratio over that which would result if the information were sent directly using the eight sequences of three binary digits. We are taking four binary digits to send three bits of information, but we are using only half of the available sequences of the 4-bit code. This is in accordance with Shannon's channel capacity theorem which allows an increase in the bandwidth for a decrease in the signal-to-noise ratio at the same information rate.

The important characteristics of either of these two code groups is that each set has orthonormal (Reference 6) properties; that is, the eight sequences in any one of the code groups are either normal, orthogonal or biorthogonal to each other. These desirable properties are expressed by the relation:

$$\frac{n}{T} \int_0^T f_l(t) f_m(t)\, dt \begin{cases} = n \text{ for } l = m & \text{(normal)} \\ = o \text{ for } l \neq m & \text{(orthogonal)} \\ = -n \text{ for } l = -m & \text{(biorthogonal)} \end{cases} \tag{2}$$

where

$f_l(t)$ = waveform of the code word,
$f_m(t)$ = the stored waveform in the m^{th} correlation detector,
n = number of digits in a code word, and
T = time length of the code word.

This integral has an interesting form in that its value is the same as the voltage output of a correlation detector. The code word $f_l(t)$ is the transmitted waveform with a *zero* represented by a signal of +1 volt and a *one* represented by -1 volt. The stored waveform $f_m(t)$ is also made up of *zeros* and *ones* represented by +1 volt and -1 volt, respectively. A table can be made up showing the voltage outputs of a set of correlation detectors of stored waveform $f_m(t)$ correlated with a set of signal waveforms $f_l(t)$.

Figure 2 shows this correlation by means of the integral applied to the 4-bit case. When the signal and the stored waveform match, +4 volts appears on the output of that detector. Putting the complement into a detector gives -4 volts. The rest of the signals when correlated with the waveforms of the same group are orthogonal and give 0 volts as the output. A maximum-likelihood detector would look at the outputs of the eight detectors in one code group and choose the "greatest of" signal. The separation between the correct signal and the closest incorrect signal is +4 volts in any one code group. If sequences are used from both code groups, the separation may only be +2 volts, as is indicated in the lower left and upper right portion of Figure 2. Noise could more easily perturb the "greatest of" detector and cause it to give a wrong indication; therefore, the least probability of error will occur if only the codes in one of the code groups are used.

Tables of the type in Figure 2 for 8-bit codes have been constructed by using a digital computer. Sixteen unique orthonormal code groups result, with each set containing sixteen characters. For a 16-bit binary code there are an extremely large number of different orthonormal 32-character code groups, any one of which could be used to encode five bits of information with a signal-to-noise improvement over the standard uncoded 5-bit binary code. A recent flight of an Air Force Blue Scout rocket carried the Digilock (References 7 and 8) coded binary PCM telemeter. The system used one of the many available 16-bit codes to encode five bits of information, exchanging bandwidth for an allowed decrease in the transmitter power for the same error rate.

	$f_l(t)$	GROUP I								GROUP II							
$f_m(t)$		0000	0011	0101	0110	1001	1010	1100	1111	0001	0010	0100	0111	1000	1011	1101	1110
GROUP I	0000	**+4**	0	0	0	0	0	0	**−4**	+2	+2	+2	−2	+2	−2	−2	−2
	0011	0	**+4**	0	0	0	0	**−4**	0	+2	+2	−2	+2	−2	+2	−2	−2
	0101	0	0	**+4**	0	0	**−4**	0	0	+2	−2	+2	+2	−2	−2	+2	−2
	0110	0	0	0	**+4**	**−4**	0	0	0	−2	+2	+2	+2	−2	−2	−2	+2
	1001	0	0	0	**−4**	**+4**	0	0	0	+2	−2	−2	−2	+2	+2	+2	−2
	1010	0	0	**−4**	0	0	**+4**	0	0	−2	+2	−2	−2	+2	+2	−2	+2
	1100	0	**−4**	0	0	0	0	**+4**	0	−2	−2	+2	−2	+2	−2	+2	+2
	1111	**−4**	0	0	0	0	0	0	**+4**	−2	−2	−2	+2	−2	+2	+2	+2
GROUP II	0001	+2	+2	+2	−2	+2	−2	−2	−2	**+4**	0	0	0	0	0	0	**−4**
	0010	+2	+2	−2	+2	−2	+2	−2	−2	0	**+4**	0	0	0	0	**−4**	0
	0100	+2	−2	+2	+2	−2	−2	+2	−2	0	0	**+4**	0	0	**−4**	0	0
	0111	−2	+2	+2	+2	−2	−2	−2	+2	0	0	0	**+4**	**−4**	0	0	0
	1000	+2	−2	−2	−2	+2	+2	+2	−2	0	0	0	**−4**	**+4**	0	0	0
	1011	−2	+2	−2	−2	+2	+2	−2	+2	0	0	**−4**	0	0	**+4**	0	0
	1101	−2	−2	+2	−2	+2	−2	+2	+2	0	**−4**	0	0	0	0	**+4**	0
	1110	−2	−2	−2	+2	−2	+2	+2	+2	**−4**	0	0	0	0	0	0	**+4**

Figure 2 —Four-bit word correlation table

If the frequencies in a PFM signal are restricted to an integral number of cycles in the pulse, PFM falls into the same class as coded binary PCM. Each pulse can be considered to contain a string of binary digits alternating between *zero* and *one* where *zero* is a signal of +1 volt and a *one* is -1 volt as before. If the pulse frequency were 6.4 kc in a 10-msec pulse, there would be the equivalent of 64 *zeros* alternating with 64 *ones*. A correlation detector, presented with this sequence, could not detect a difference between the PFM pulse and a binary pulse code modulation word 128 bits in length. If the bit rate were changed to 130 bits in the same pulse length, this would be equivalent to a pulse frequency of 6.5 kc. Application of Equation 2 with $f_l(t)$ equal to the waveform of the 6.5 kc pulse and $f_m(t)$ equal to the 6.4 kc waveform in the correlation detector yields zero as the cross-correlation. An orthogonal code set where the frequencies of the pulse instead of the binary word are used as coordinates can be made up in the same manner as Figure 2. This is shown in Figure 3.

Figure 3 uses the alternating *zeros* and *ones* code for a word length of from 100 to 300 bits. In effect, this code set is composed of only one binary word taken from every other table like Figure 2 from $n = 100$ to $n = 300$. Other tables that will have orthogonal properties can be constructed by selecting another sequence or character from each of the

FREQUENCY (kc)

FREQUENCY (kc)	5.0	5.1	5.2	5.3				14.7	14.8	14.9	15.0
5.0	**100**	0	0	0	•	•	•	0	0	0	0
5.1	0	**102**	0	0	•	•	•	0	0	0	0
5.2	0	0	**104**	0	•	•	•	0	0	0	0
5.3	0	0	0	**106**	•	•	•	0	0	0	0
	•	•	•	•	•	•	•	•	•	•	•
	•	•	•	•	•	•	•	•	•	•	•
	•	•	•	•	•	•	•	•	•	•	•
14.7	0	0	0	0	•	•	•	**294**	0	0	0
14.8	0	0	0	0	•	•	•	0	**296**	0	0
14.9	0	0	0	0	•	•	•	0	0	**298**	0
15.0	0	0	0	0	•	•	•	0	0	0	**300**

Figure 3—Pulse frequency modulation correlation table

constant-bit-number tables. However, implementing these code words would be very difficult. Pulse frequency modulation uses the alternating *zero* and *one* code words which are extremely easy to generate.

The odd values of n between 100 and 300 could also have been used in constructing Figure 3. The odd values are orthogonal to each other and to the even values. Use of the odd values will double the number of sequences; however, a noncoherent detector has difficulty in separating adjacent codes. If the odd values are not used, there is some advantage in using noncoherent detectors. First, it is easier to construct a large number of noncoherent detectors than the coherent type. A simple bandpass filter of proper width is a form of noncoherent detector for an alternating string of *zeros* and *ones*. Secondly, frequencies other than integral frequencies are permitted. This is necessary if analog oscillators are to be used in the telemeter; otherwise, the signal must be quantized at the source.

A set of contiguous filters can be used as a substitute for the maximum-likelihood coherent detectors. Each filter matches one of the pulse frequencies so that adjacent pulses do not excite the filter. With realizable filters, however, the adjacent pulses do excite the filter to some degree. This causes an occasional slight reduction in the accuracy since an adjacent filter may be triggered by the sum of the cross-correlation output and the noise.

With maximum-likelihood coherent detectors, code words with alternating *ones* and *zeros* may also be used. This is equivalent to a biorthogonal code set since alternating *ones* and *zeros* are the complement of alternating *zeros* and *ones*. Shifting the phase of the pulses by 180 degrees produces the complement.

The use of contiguous filters in the detector instead of coherent detectors will cause a 3-db deterioration in the signal-to-noise ratio since quadrature noise is not eliminated. This penalty is offset by the fact that the analog oscillators allow frequencies between the integral values so that during good signal-to-noise conditions the precision of measurement can be increased. In poor signal-to-noise conditions it is only possible to detect the presence or absence of the pulse in one of the filters. If there are 2^7 filters, the accuracy is still better than 1 percent.

Viterbi (Reference 9) has recently published curves showing the error probability as a function of the ratio:

$$\frac{\text{received signal energy/bit}}{\text{noise power/unit bandwidth}} .$$

These curves are plotted for both orthogonal and biorthogonal codes for various values of n, where n is the number of coded bits. For a PFM system with integral values of frequency which uses 128 coherent detectors, the value of n would be 7. If the pulse frequency range were doubled, requiring 256 detectors, the value of n would be 8. These curves can be directly used to evaluate a PFM signal. Explorer XII used 100 filters. If they had been coherent detectors, n would be $\log_2 100 = 6.64$. This is equivalent to a coded binary phase-coherent system with $n = 6.64$.

In summary, the PFM code set is made up of selected sequences from many coded binary sets. With the transmission of analog frequencies, the contiguous filters may be substituted for the coherent detectors and greater precision can be attained under poor signal-to-noise conditions.

DESIGN

Figure 4 pictures a group of standard PFM modules assembled on a printed-wiring board for use in the first International Ionosphere Satellite (United Kingdom No. 1) telemeter. These standard welded modules were developed at the Goddard Space Flight Center. Besides the standard analog and digital pulsed oscillator modules there is an eight-input-gate module to subcommutate eight channels into one analog oscillator, and a six-input-gate switching module for switching eighteen bits of information into a digital oscillator

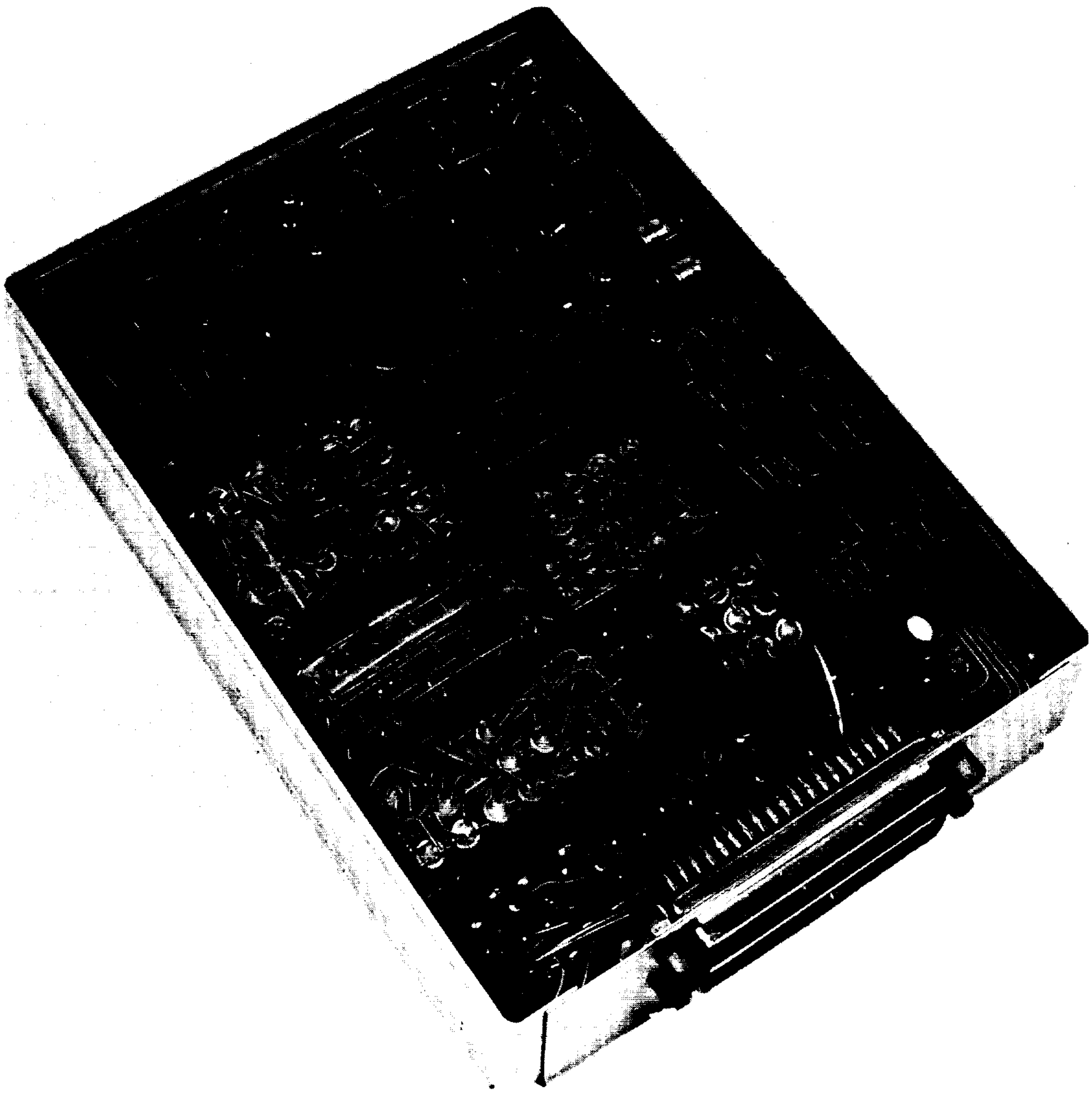

Figure 4—PFM telemetry encoder showing modular construction

three bits at a time. With these units as building blocks it is possible to build a high-efficiency telemeter quickly and with a minimum of effort.

The importance of circuitry which minimizes power drain cannot be overemphasized. The present design of analog and digital oscillators requires only 2 to 4 mw of power to operate. The binary stages used in the switching logic use less than 1 mw for each stage. Complex systems built from these low-power-drain modules use only a small percentage of the available satellite dc power.

PULSE FREQUENCY MODULATION DATA ACQUISITION EQUIPMENT

Reception of PFM signals at maximum range requires the use of phase-coherent detection equipment for the carrier. For Explorer XII, this was accomplished by translating the last intermediate-frequency output of the Mark II Minitrack receiver directly into an Interstate Mod VIII tracking filter. The coherent phase modulation output was recorded directly on magnetic tape along with timing signals. The tapes were sent back to the Goddard Space Flight Center for data reduction. A complete description of the data acquisition equipment will be found in an article by Creveling, Ferris and Stout (Reference 10).

At the data reduction center, the magnetic tapes are played back into a bank of contiguous filters. These filters were developed under NASA contract by Interstate Electronics Corporation. The bank consists of 120 bandpass filters, each with a bandwidth of 100 cps at its 3-db point. The filters are contiguous in that their spacings are 100 cps between centers. They cover the frequency range from 4 to 16 kc. Since only one pulse frequency is present at a time, only one of the 120 filters will be excited. A decision circuit (sometimes called an auction circuit) decides which filter has the greatest signal; all other filters are biased off, and the one filter is gated to the output. The output signal-to-noise is therefore the signal-to-noise in the 100 cps filter. With a reasonable signal-to-noise ratio, the filter output can be passed on to a discriminator for making more precise measurements (Reference 11). The magnetometer data of Explorer X were read to a precision of nine bits and sometimes to ten bits, depending on the signal-to-noise; at maximum range (150,000 miles) the measurement accuracy was maintained at 1 percent. For the general usage of PFM, the discriminator is not used and the signal is quantized into one of 100 values by recording only the number of the filter which contained the greatest signal. This filter number is later used with the experiment calibration data in computing the final data.

CONCLUSIONS

Pulse frequency modulation provides an easy method for generating an orthogonal or near-orthogonal code set. The use of such a set transmitted from a satellite or spacecraft will result in a reduction in the transmitter power requirement for the same error rate over that of uncoded binary pulse code modulation. The state of the art is such that off-the-shelf modules can be used to easily build an efficient telemeter, tailored to any combination of channel assignments. For applications at maximum range with the requirement for low weight and low power drain, PFM can perform most satisfactorily.

REFERENCES

1. Wolf, J. K., "Comparison of N-ary Transmission Systems," Rome Air Development Center, Tech. Note 60-210, December 1960, p. 10

2. Kotel'nikov, V. A., "The Theory of Optimum Noise Immunity," New York: McGraw-Hill, 1959, p. 124

3. Shannon, C. E., "A Mathematical Theory of Communication," *Bell Sys. Tech. J.* 27(3): 379-423, July 1948; 27(4):623-656, October 1948

4. Hamming, R. W., "Error Detecting and Error Correcting Codes," *Bell Sys. Tech. J.* 29(2):147-160, April 1950

5. Reed, I. S., "A Class of Multiple-Error-Correcting Codes and the Decoding Scheme," *Trans. IRE Prof. Group on Information Theory,* PGIT-4:38-49, September 1954

6. Lee, Y. W., "Statistical Theory of Communication," New York: John Wiley and Sons, 1960, p. 460

7. Sanders, R. W., "Digilock Telemetry System" in: *Proc. Nat. Sympos. on Space Electronics and Telemetry,* San Francisco, September 1959, New York: IRE Prof. Group on Space Electronics and Telemetry, 1959, Paper 6.3

8. Sanders, R. W., "Communication Efficiency Comparison of Several Communication Systems," *Proc. IRE* 48(4):575-588, April 1960

9. Viterbi, A. J., "On Coded Phase-Coherent Communications," *IRE Trans. on Space Electronics and Telemetry,* SET-7(1):3-14, March 1961

10. Creveling, C. J., Ferris, A. G. and Stout, C. M., "Automatic Data Processing," *IRE Trans. on Space Electronics and Telemetry* (To be published)

11. Rochelle, R. W., "Signal-to-Noise Considerations for a Space Telemetry System," *Proc. IRE* 48(4):691-693, April 1960